



Multi-Task Learning via Generalized Tensor Trace Norm

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Introduction

01

02

Existing tensor trace norms

Generalized Tensor Trace
Norm (GTTN)

03

04

Experiments



Multi-Task Learning via Generalized Tensor Trace Norm

01 Introduction

Introduction

Multi-Task Learning



Human Learning



Learn multiple tasks **simultaneously**



Use the knowledge learned in a task to **help** the learning of another task



play tennis



play squash

Introduction

Multi-Task Learning

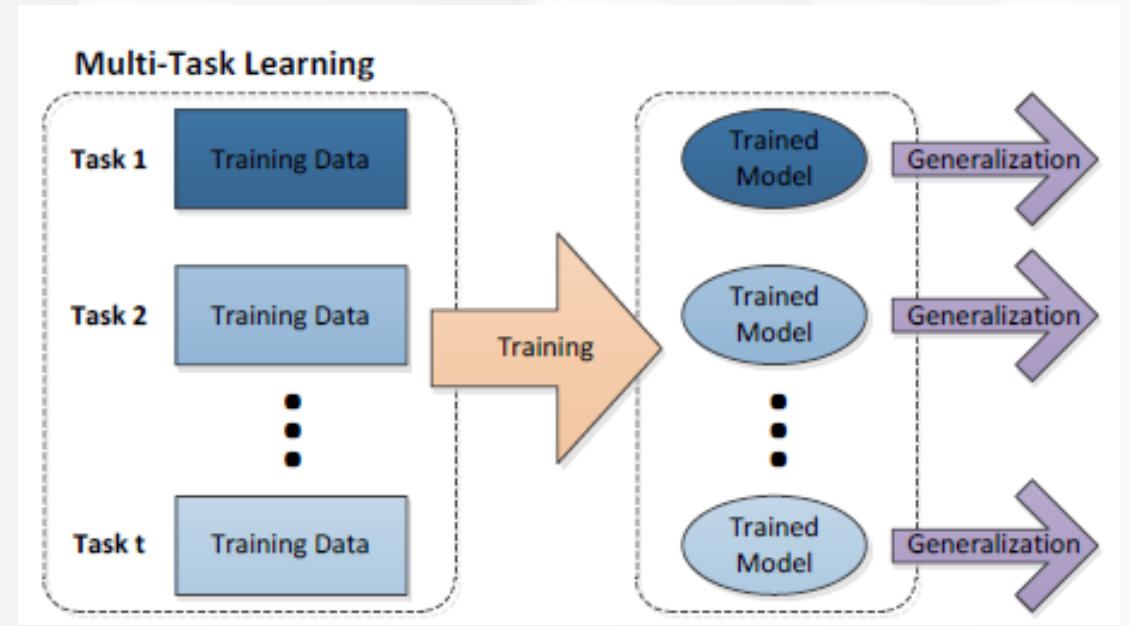
Learn multiple related tasks **jointly**



The knowledge contained in a task can be **leveraged** by other tasks



Improve the generalization performance of all the tasks



Introduction

Multi-Task Learning in Natural Language Processing

arXiv.org > cs > arXiv:2109.09138

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Computer Science > Artificial Intelligence

[Submitted on 19 Sep 2021]

Multi-Task Learning in Natural Language Processing: An Overview

Shijie Chen, Yu Zhang, Qiang Yang

Deep learning approaches have achieved great success in the field of Natural Language Processing (NLP). However, deep neural models often suffer from overfitting and data scarcity problems that are pervasive in NLP tasks. In recent years, Multi-Task Learning (MTL), which can leverage useful information of related tasks to achieve simultaneous performance improvement on multiple related tasks, has been used to handle these problems. In this paper, we give an overview of the use of MTL in NLP tasks. We first review MTL architectures used in NLP tasks and categorize them into four classes, including the parallel architecture, hierarchical architecture, modular architecture, and generative adversarial architecture. Then we present optimization techniques on loss construction, data sampling, and task scheduling to properly train a multi-task model. After presenting applications of MTL in a variety of NLP tasks, we introduce some benchmark datasets. Finally, we make a conclusion and discuss several possible research directions in this field.

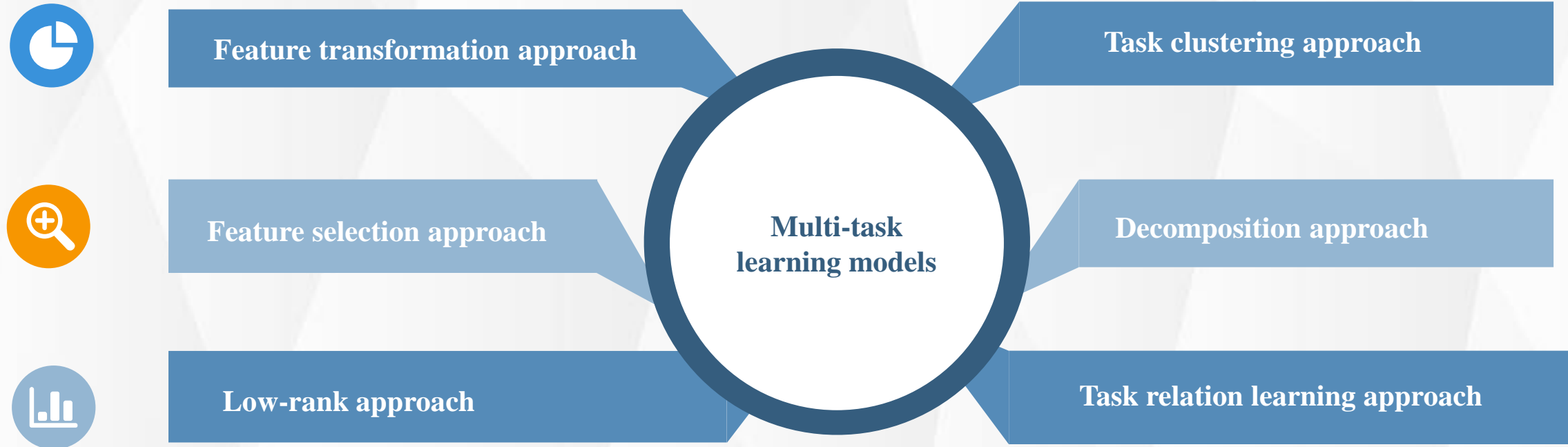
Subjects: **Artificial Intelligence (cs.AI)**

Cite as: [arXiv:2109.09138](#) [cs.AI]

(or [arXiv:2109.09138v1](#) [cs.AI] for this version)

Shijie Chen, Yu Zhang, Qiang Yang. Multi-Task Learning in Natural Language Processing: An Overview. arXiv:2109.09138, 2021.

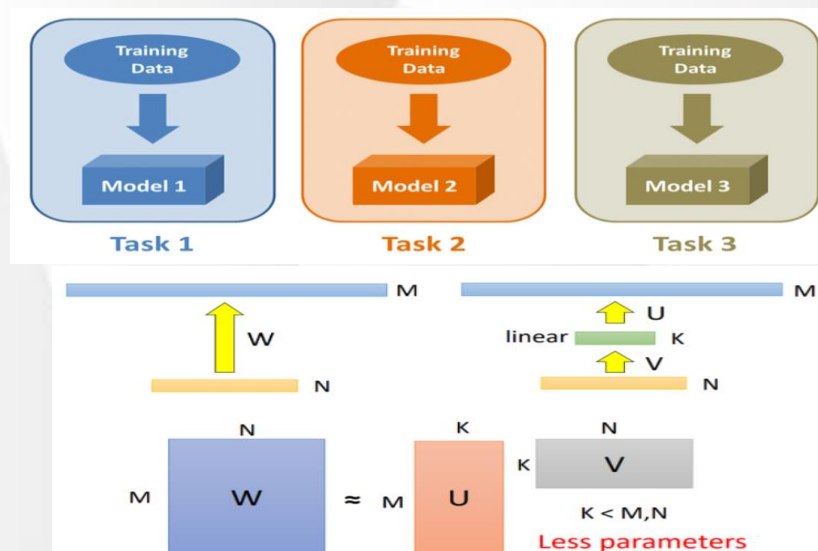
Introduction



Yu Zhang and Qiang Yang, A Survey on Multi-Task Learning, IEEE TKDE 2021

Introduction

Low-rank approach

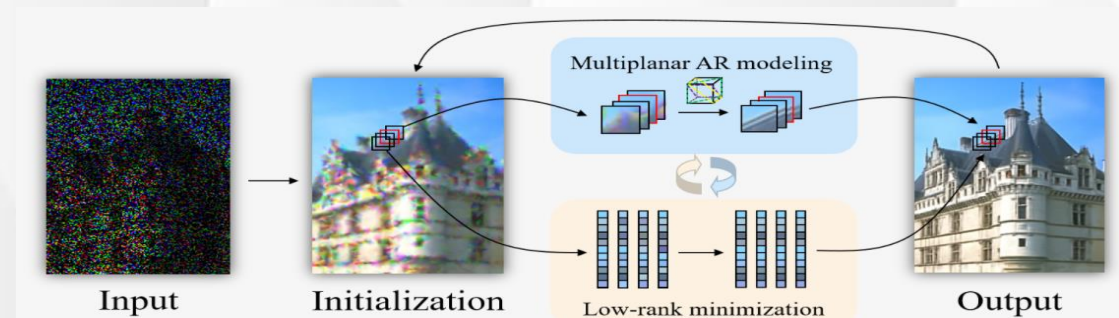
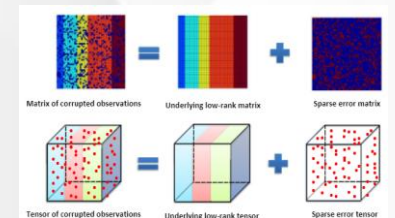


Low-rank approach

Relatedness among multiple tasks



Low-rank of parameters



Introduction

Low-rank approach

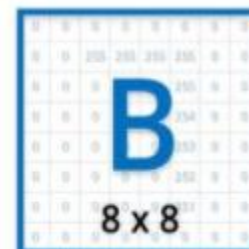
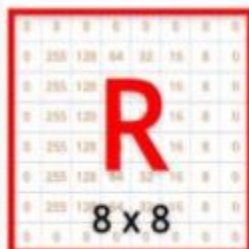
Matrix parameters



Matrix trace norm

Multi-task

Image

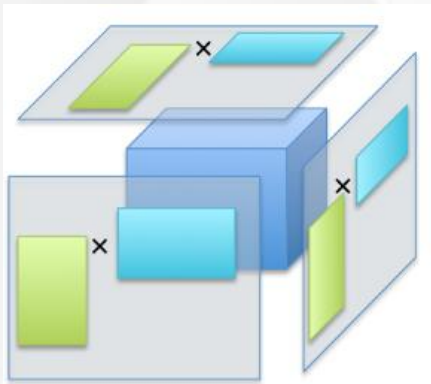


Tensor trace norm

Multi-class classification

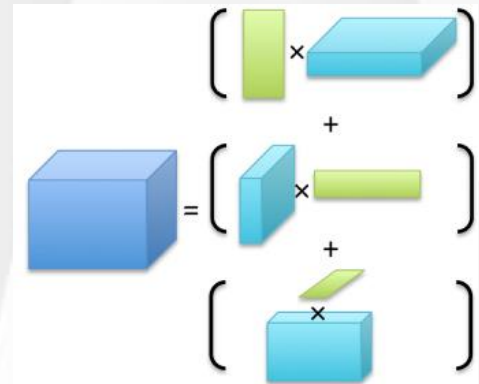
Introduction

Overlapped tensor trace norms



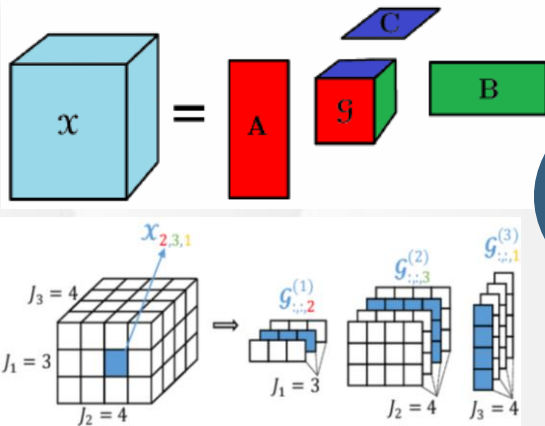
Tensor trace norm

Latent tensor trace norms



Introduction

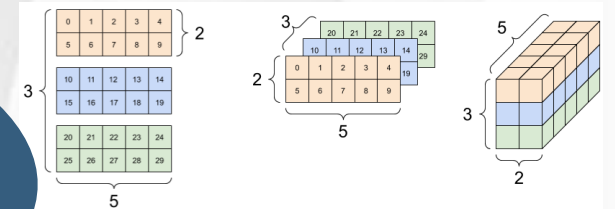
Tucker Trace Norm



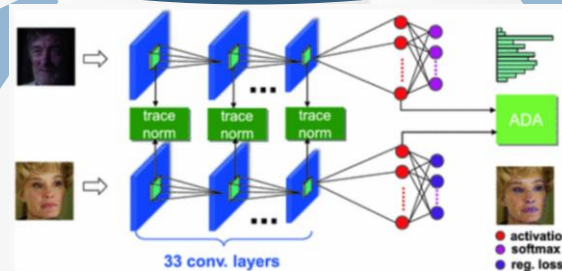
Tensor-Train (TT) Trace Norm

Overlapped tensor trace norms

LAF Trace Norm



Tensor-Ring (TR) Trace Norm





Multi-Task Learning via Generalized Tensor Trace Norm

02 Existing tensor trace norms

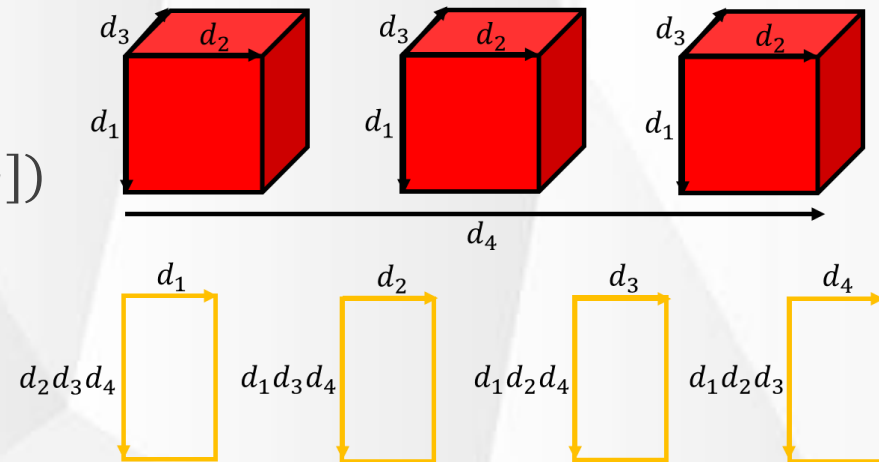
Existing tensor trace norms

Tucker Trace Norm

$$|||\mathcal{W}|||_* = \sum_{i=1}^p \alpha_i ||\mathcal{W}_{(i)}||_* \quad \mathcal{W} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

$$\text{s.t.} \begin{cases} \mathcal{W}_{(i)} := \text{reshape}(\text{permute}(\mathcal{W}, [i, 1, \dots, i-1, i+1, \dots, p]), [d_i, \prod_{j \neq i} d_j]) \\ \alpha_i \geq 0, \sum_{i=1}^p \alpha_i = 1 \end{cases}$$

4-way tensor

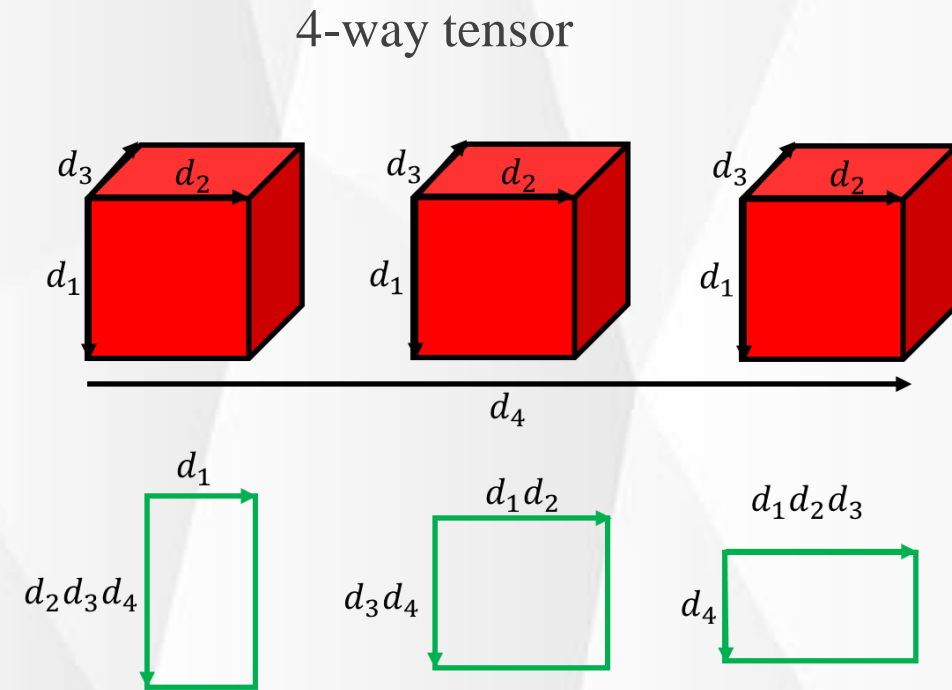


Existing tensor trace norms

TT Trace Norm

$$|||\mathcal{W}|||_* = \sum_{i=1}^{p-1} \alpha_i ||\mathcal{W}_{[i]}||_* \quad \mathcal{W} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

$$\text{s.t.} \begin{cases} \mathcal{W}_{[i]} := \text{reshape}(\mathcal{W}, [\prod_{j=1}^i d_j, \prod_{j=i+1}^p d_j]) \\ \alpha_i \geq 0, \sum_{i=1}^p \alpha_i = 1 \end{cases}$$



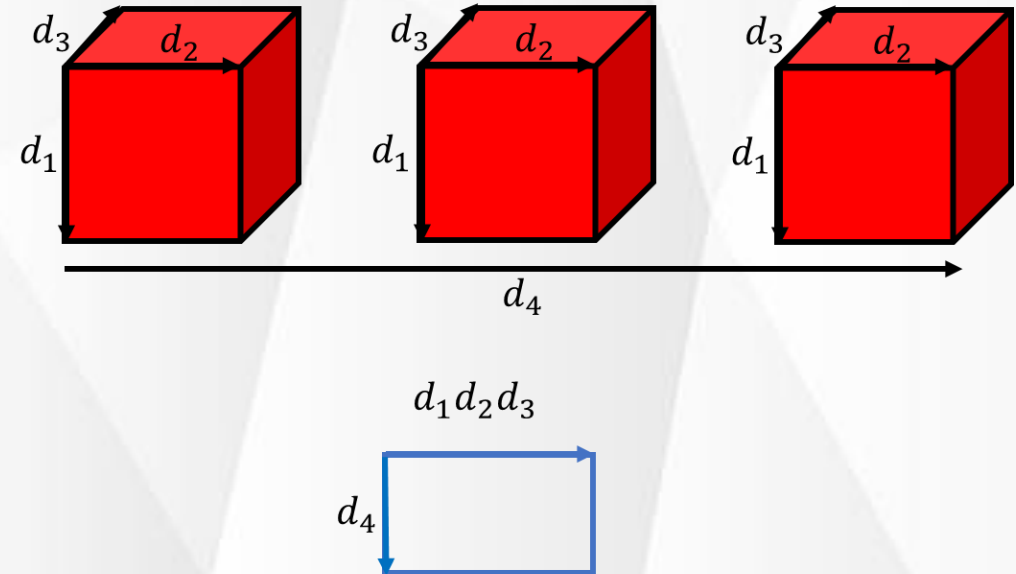
Existing tensor trace norms

LAF Trace Norm

$$|||\mathcal{W}|||_* = ||\mathcal{W}_{(p)}||_* \quad \mathcal{W} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

$$\text{s.t. } \mathcal{W}_{(p)} := \text{reshape}(\mathcal{W}, [\prod_{j=1}^{p-1} d_j, d_p])$$

4-way tensor



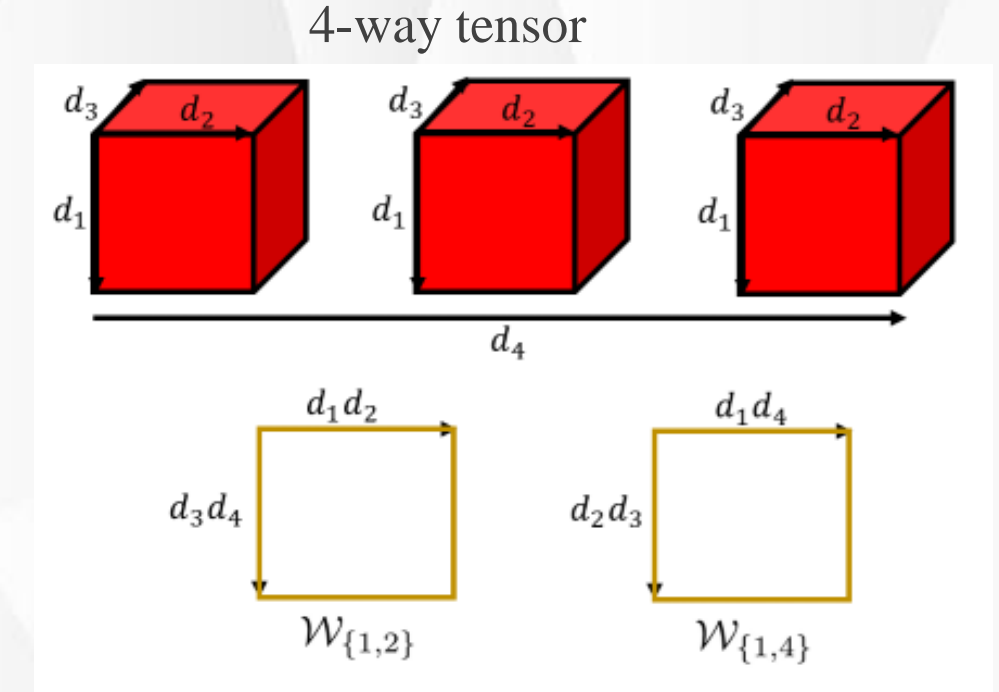
Existing tensor trace norms

TR trace norm

$$|||\mathcal{W}|||_* = \sum_{i=1}^p \alpha_i ||\mathcal{W}_{\langle i,d \rangle}||_*$$

$$\mathcal{W} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

$$\text{s.t.} \quad \left\{ \begin{array}{l} \mathcal{W}_{\langle i,d \rangle} := \text{reshape}(\text{permute}(\mathcal{W}, [t, \dots, i, i+1, \dots, t-1]), [\prod_{j=t}^i d_j, \prod_{j=i+1}^{t+1} d_j]) \\ t = \begin{cases} i - d + 1 & \text{if } d \leq i \\ i - d + 1 + p & \text{otherwise} \end{cases} \end{array} \right.$$



$$\alpha_i \geq 0, \sum_{i=1}^p \alpha_i = 1$$



Multi-Task Learning via Generalized Tensor Trace Norm

03 Generalized Tensor Trace Norm (GTTN)

Generalized Tensor Trace Norm (GTTN)

How to choose the way of tensor flattening?

- ◆ Try all possible ways of tensor flattening.



Analysis on Existing Tensor Trace Norms

Given the way of tensor flattening, how to determine the importance of resultant tensor flattenings?

Learn Weights:

- ◆ Variable
- ◆ Minimum
- ◆ Maximum
- ◆ Meta-learning



Generalized Tensor Trace Norm (GTTN)

How to choose the way of tensor flattening?

→ Try all possible ways of tensor flattening

$$\begin{aligned} \mathcal{W}_* &= \sum_{\mathbf{s}} \alpha_{\mathbf{s}} \|\mathcal{W}_{\{\mathbf{s}\}}\|_* & \mathcal{W} &\in \mathbb{R}^{d_1 \times \dots \times d_p} \\ \text{s.t. } \left\{ \begin{array}{l} \mathcal{W}_{\{\mathbf{s}\}} := \text{reshape}(\text{permute}(\mathcal{W}, [\mathbf{s}, \neg\mathbf{s}]), [\prod_{i \in \mathbf{s}} d_i, \prod_{j \in \neg\mathbf{s}} d_j]) \\ \alpha_{\mathbf{s}} \geq 0, \sum_{\mathbf{s}} \alpha_{\mathbf{s}} = 1 \end{array} \right. \end{aligned}$$

Generalized Tensor Trace Norm (GTTN)

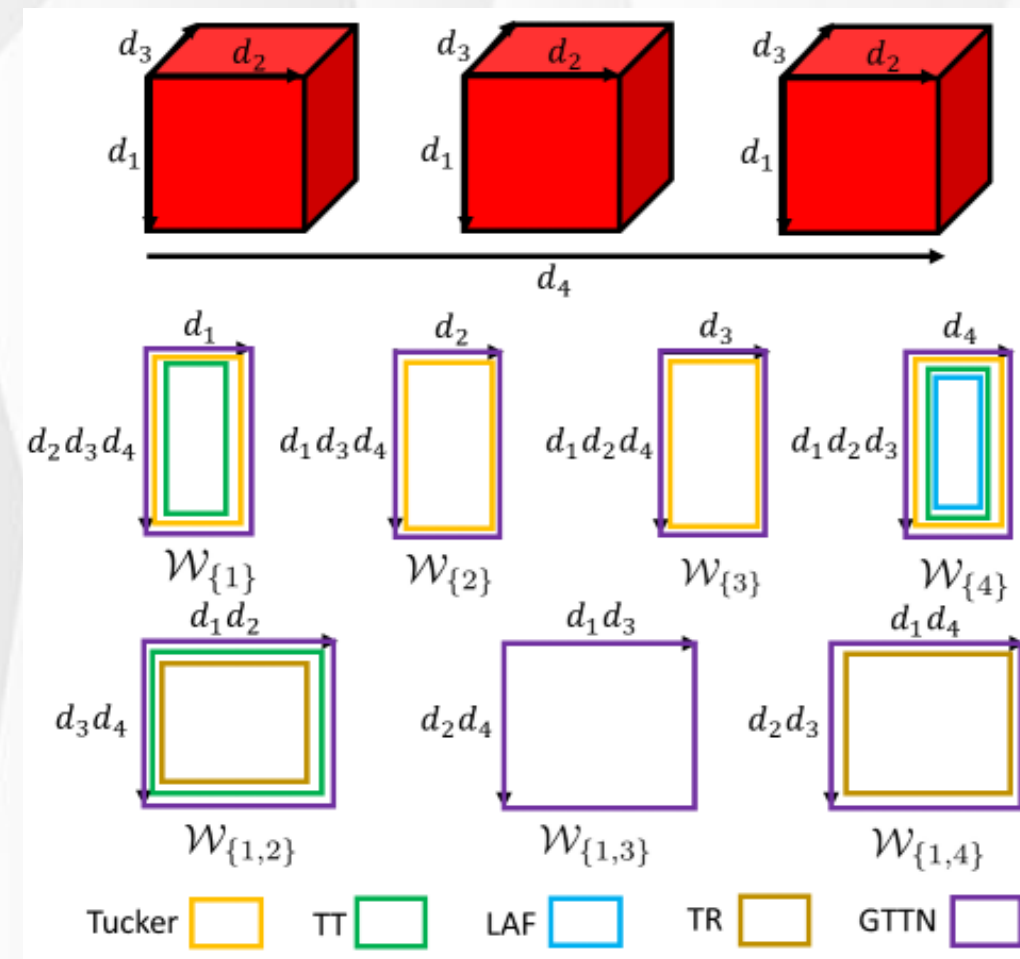
How to choose the way of tensor flattening?

◆ Lemma 1: For a p -way tensor, There are $2^{p-1}-1$ distinct tensor flattening.

◆ $p \leq 5$

◆ # distinct tensor flattening ≤ 15

Comparison among the Tucker trace norm, TT trace norm, LAF trace norm, TR trace norm with $d=2$, and GTTN for a 4-way tensor.



distinct tensor flattening: 4 3 1 2 7

Generalized Tensor Trace Norm (GTTN)

How to determine α ?



Generalized Tensor Trace Norm (GTTN)

Learning Weights

Viewing α as variables to be optimized

$$\min_{\Theta, \alpha} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \Theta), y_j^i) + \lambda \sum_{i=1}^N \alpha_s \|\mathcal{W}_{\{s\}}\|_* \quad \text{s. t. } \alpha_s \geq 0, \sum_s \alpha_s = 1$$

$$\alpha_s = \frac{\exp\{\beta_s\}}{\sum_{t \in [p], t \neq \emptyset} \exp\{\beta_t\}}$$

softmax function

β_s instead of α_s is treated as a variable.

Generalized Tensor Trace Norm (GTTN)

Learning Weights

Optimizing minimum matrix trace norm

$$\min_{\theta} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \theta), y_j^i) + \lambda \min_{\substack{s \subseteq [p] \\ s \neq \emptyset}} \|\mathcal{W}_{\{s\}}\|_*$$

Mathematically



Equivalent

$$\min_{\theta, \alpha} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \theta), y_j^i) + \lambda \sum_{s=1}^N \alpha_s \|\mathcal{W}_{\{s\}}\|_* \quad \text{s.t. } \alpha_s \geq 0, \sum_s \alpha_s = 1$$

softmax function

cannot achieve 0 or 1 exactly

Generalized Tensor Trace Norm (GTTN)

Learning Weights

Optimizing maximum matrix trace norm

$$\min_{\Theta} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \Theta), y_j^i) + \lambda \max_{\substack{s \in [p] \\ s \neq \emptyset}} \|\mathcal{W}_{\{s\}}\|_*$$

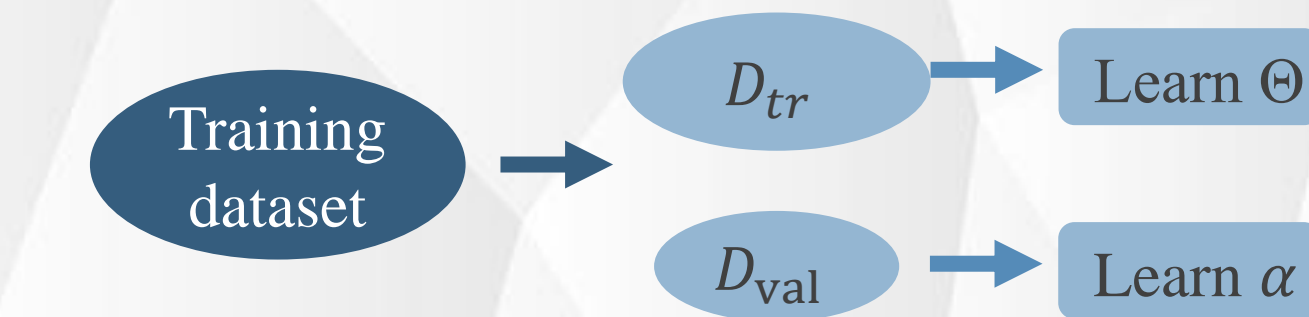
Pay attention to flattenings with the **largest** trace norm

Penalize all the matrix trace norm

Generalized Tensor Trace Norm (GTTN)

Learning Weights

Meta-learning method



$$\min_{\alpha} \sum_{i=1}^m \frac{1}{|D_{val}^i|} \sum_{(x,y) \in D_{val}^i} l(f_i(\mathbf{x}; \Theta^*), y)$$

$$\text{s.t. } \Theta^* = \operatorname{argmin} \sum_{i=1}^m \frac{1}{|D_{tr}^i|} \sum_{(x,y) \in D_{tr}^i} l(f_i(\mathbf{x}; \Theta), y) + \lambda \sum_{s=1}^p \alpha_s ||\mathcal{W}_{\{s\}}||_*$$

Generalized Tensor Trace Norm (GTTN)

Objective function

Given the Generalized Tensor Trace Norm (GTTN), the objective function of the deep multi-task model can be expressed as:

$$\min_{\Theta} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \Theta), y_j^i) + \lambda |||\mathcal{W}|||_*$$

Empirical loss

Regularization term: GTTN

Generalized Tensor Trace Norm (GTTN)

Analysis

THEOREM 2. *For the solution $\hat{\mathcal{W}}$ of problem (8) and $\delta > 0$, with probability at least $1 - \delta$, we have*

$$L(\hat{\mathcal{W}}) \leq \hat{L}(\hat{\mathcal{W}}) + \frac{2\rho\gamma C}{mn_0} \min_{\substack{s \neq \emptyset \\ s \subset [p]}} \left(\frac{\kappa m \sqrt{\ln d_s}}{\alpha_s n_0 d} + \frac{\ln d_s}{\alpha_s n_0} \right) + \sqrt{\frac{2}{m} \ln \frac{1}{\delta}}.$$



Multi-Task Learning via Generalized Tensor Trace Norm

04 Experiments

Experiments

Baselines

DMTL	Deep Multi-Task Learning method
Tucker	The Tucker trace norm method
TT	The TT trace norm method
LAF	The LAF trace norm method
LAF- i	The trace norm regularization method based on the i -th axis flattening
TR	TR trace norm method ($d = 2$)
LAF-TF	LAF tensor factorization method
Prod	The rank-product regularization method
GTTN-a	Setting the weights in GTTN to be same

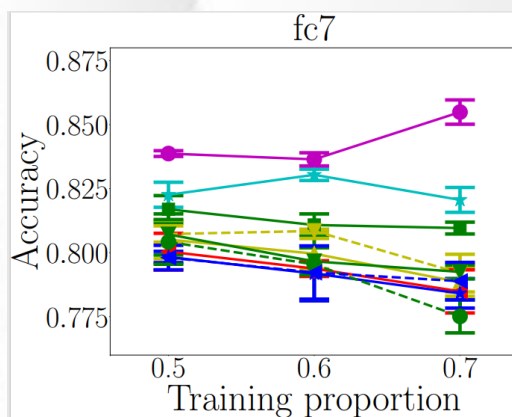
Experiments

Datasets

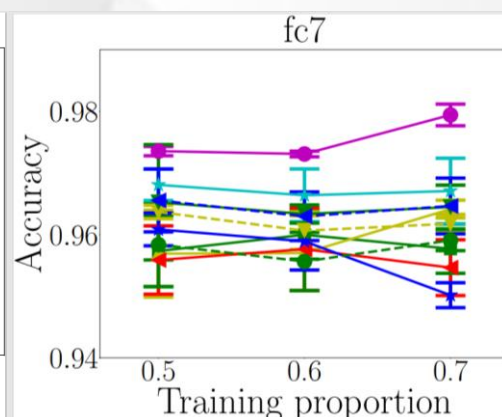
Dataset	#Images	#Classes	#Tasks
ImageCLEF	2,400	12	4
Office-Caltech	2,533	10	4
Office-31	4,110	31	3
Office-Home	15,500	65	4
DomainNet	600,000	345	6

Experiments

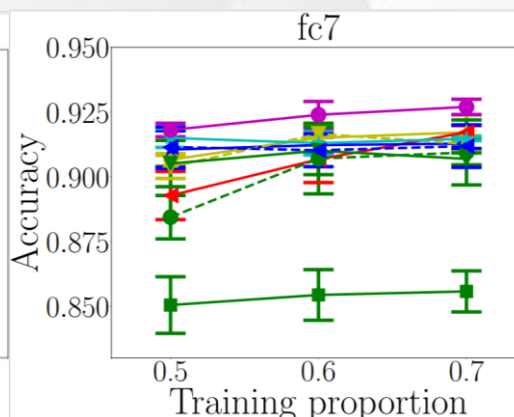
Results: fc7 layer



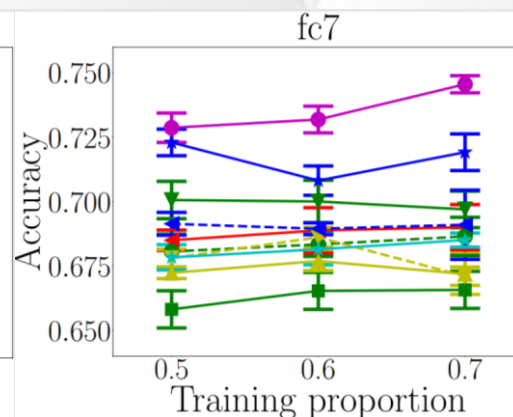
ImageCLEF



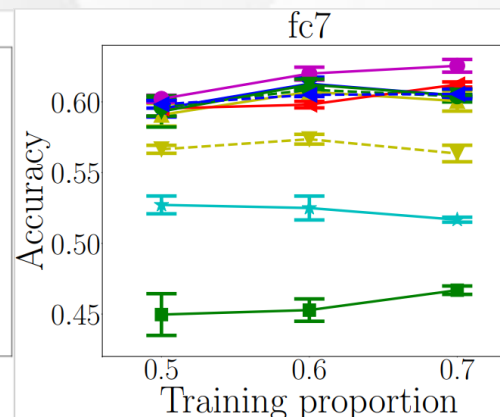
Office-Caltech



Office-31



Office-Home



DomainNet

DMTL

Tucker

TT

LAF

LAF-1

LAF-2

Prod

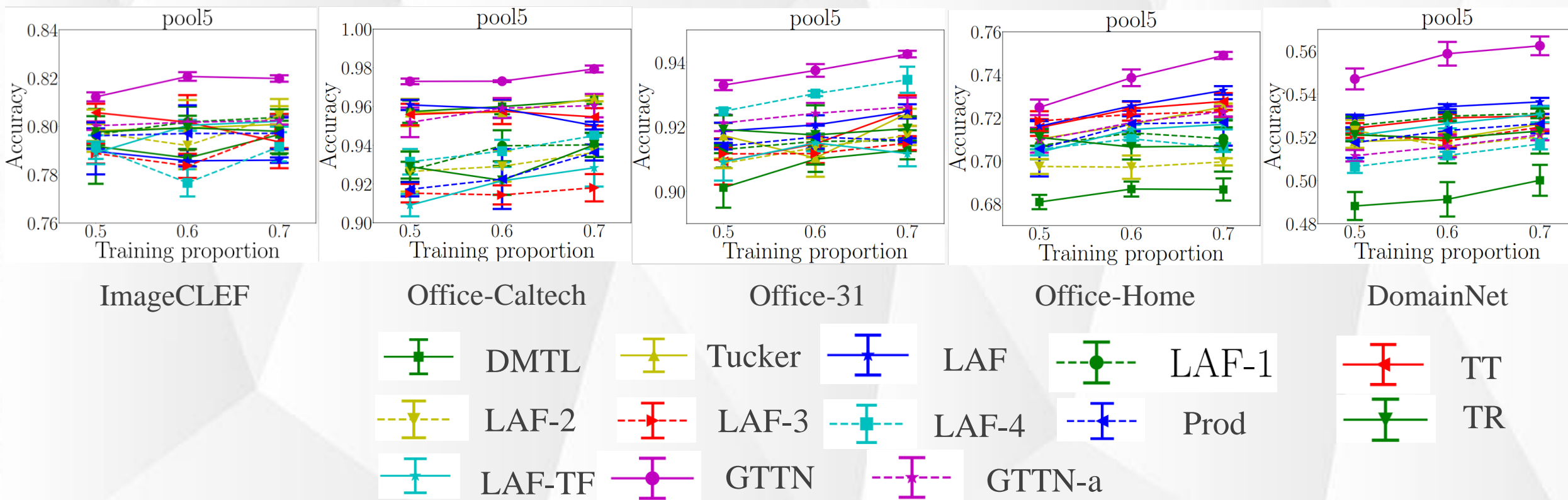
TR

LAF-TF

GTTN

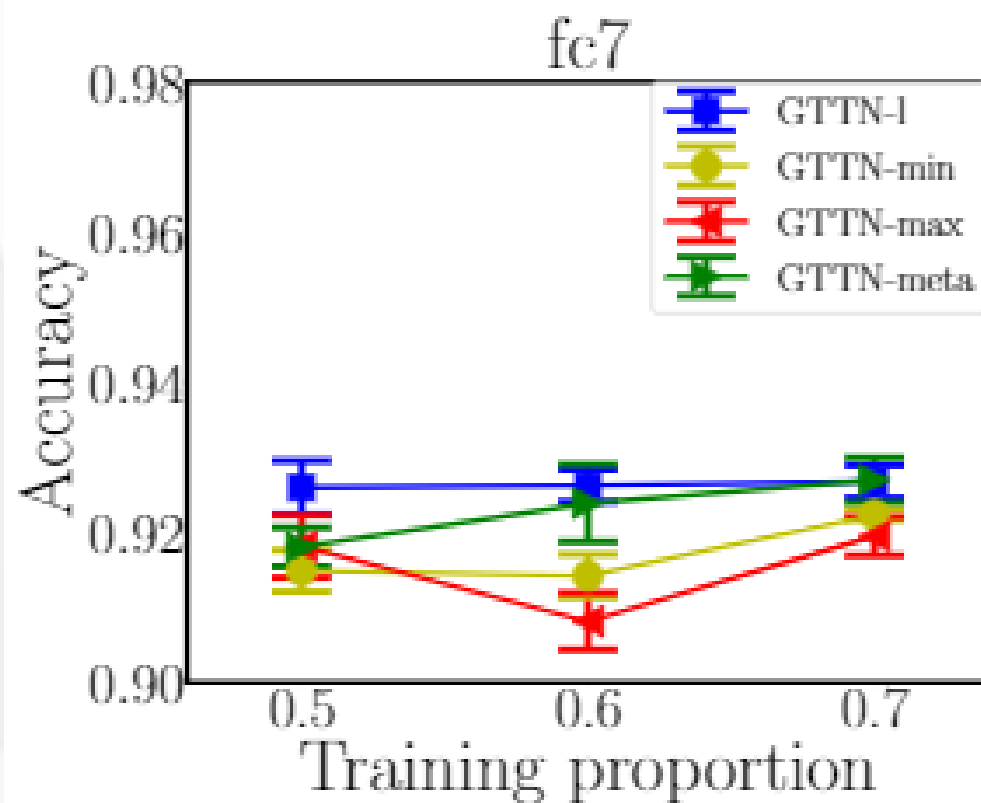
Experiments

Results: pool5 layer

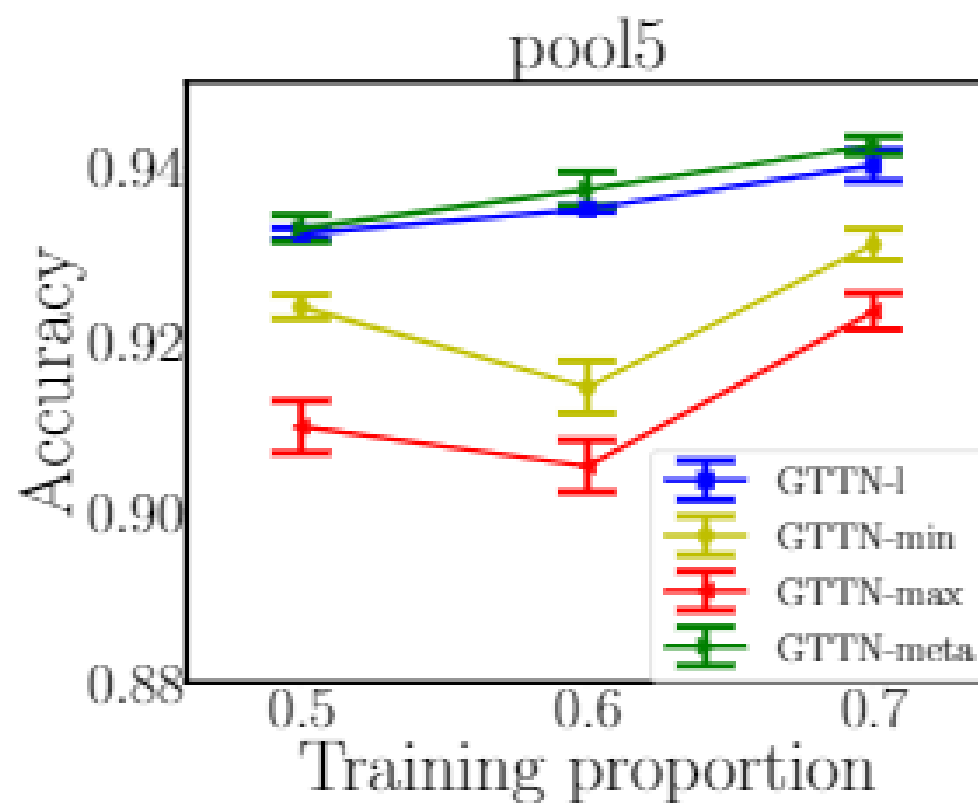


Experiments

Comparison on Strategies to Learn Weights



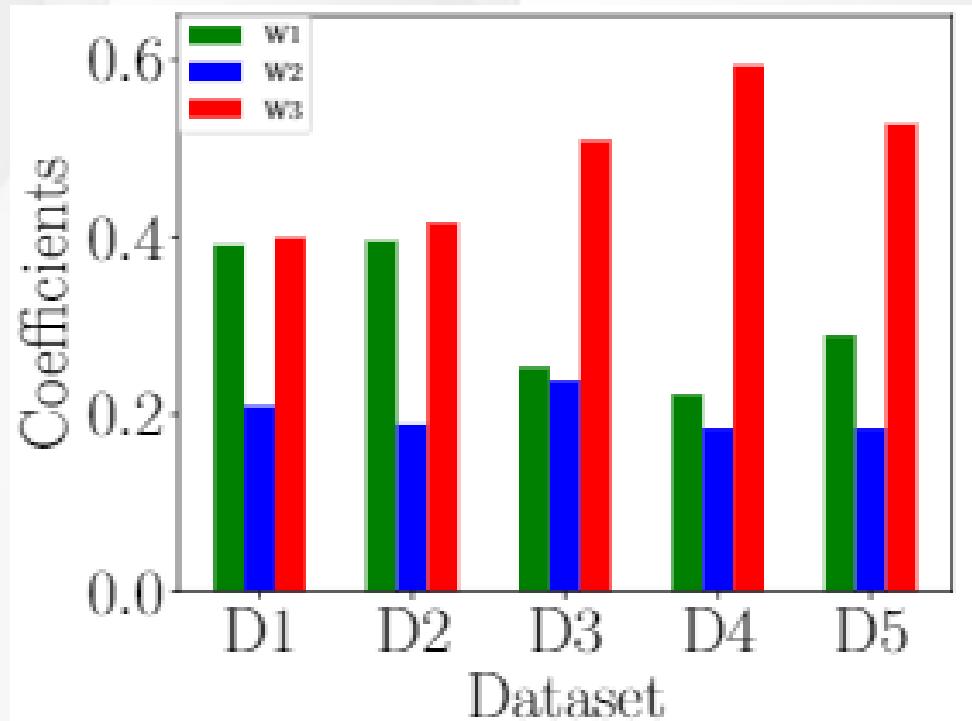
(a) Comparison of GTTN



(b) Comparison of GTTN

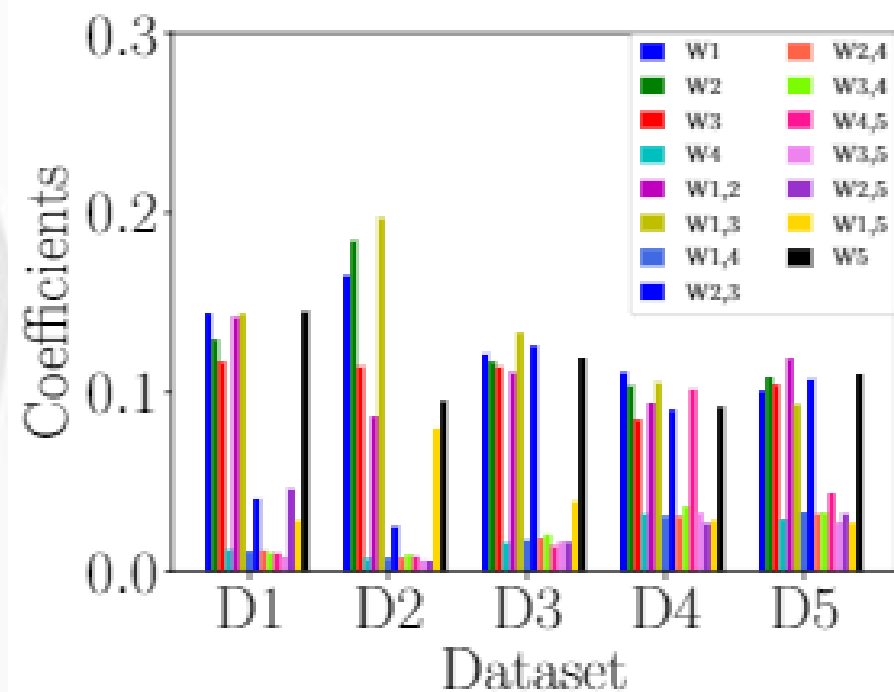
Experiments

Analysis on Learned Weights



(c) Learned α (fc7)

$w_{\{2\}}$ is smaller

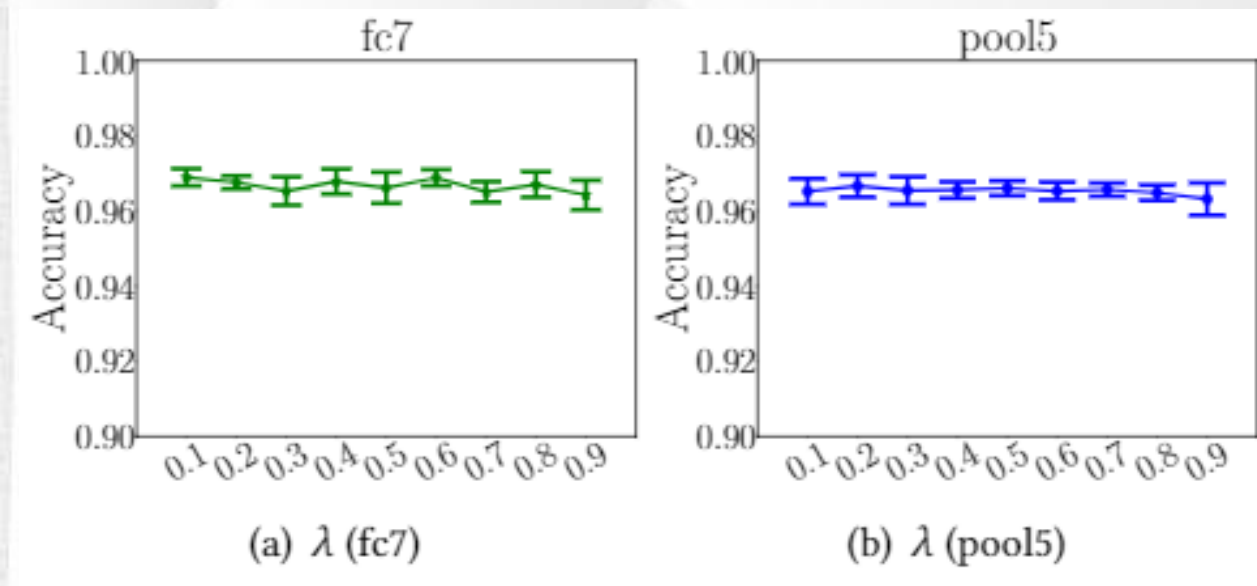


(d) Learned α (pool5)

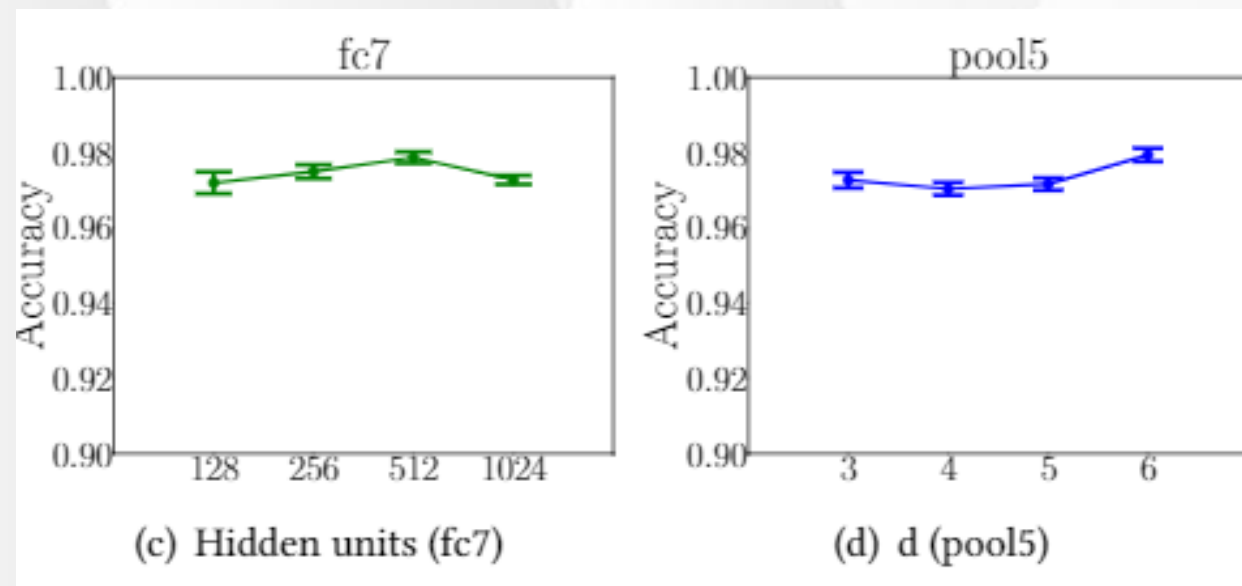
$w_{\{1\}}, w_{\{2\}}, w_{\{3\}}, w_{\{1,2\}}, w_{\{1,3\}}, w_{\{5\}}$ are larger

Experiments

Sensitivity Analysis



The performance is **not sensitive** to λ



number of hidden units = 512, $d=6$

Conclusion



- The generalized tensor trace norm (GTTN) to capture all the low-rank is effective.
- Learning weights of each tensor flattening to identify the importance of each structure is helpful.
- The GTTN method performs better than baseline methods.

Thank you !

