



# Multi-Task Learning via Generalized Tensor Trace Norm

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Introduction

01

02

Existing tensor trace norms

Generalized Tensor Trace  
Norm (GTTN)

03

04

Experiments



# Multi-Task Learning via Generalized Tensor Trace Norm

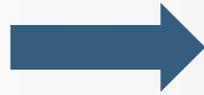
## 01 Introduction

# Introduction

## Multi-Task Learning



Human Learning



Learn multiple tasks **simultaneously**



Use the knowledge learned in a task to **help** the learning of another task



play tennis



play squash

# Introduction

## Multi-Task Learning

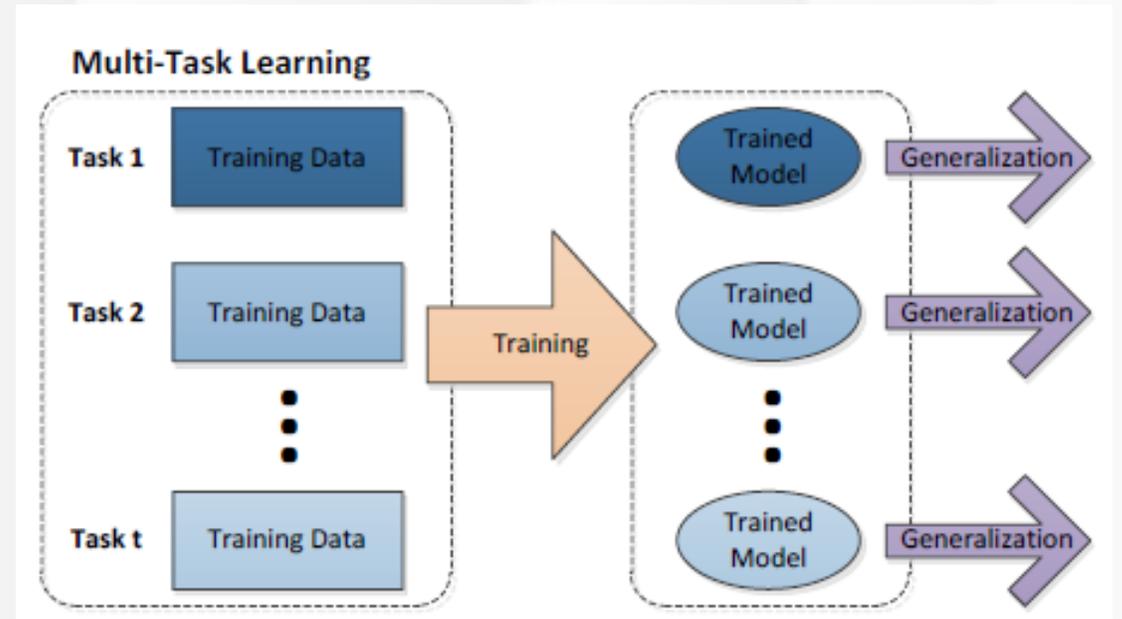
Learn multiple related tasks **jointly**



The knowledge contained in a task can be **leveraged** by other tasks



**Improve** the generalization performance of all the tasks



# Introduction

## Multi-Task Learning in Natural Language Processing

arXiv.org > cs > arXiv:2109.09138

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[Submitted on 19 Sep 2021]

### Multi-Task Learning in Natural Language Processing: An Overview

Shijie Chen, Yu Zhang, Qiang Yang

Deep learning approaches have achieved great success in the field of Natural Language Processing (NLP). However, deep neural models often suffer from overfitting and data scarcity problems that are pervasive in NLP tasks. In recent years, Multi-Task Learning (MTL), which can leverage useful information of related tasks to achieve simultaneous performance improvement on multiple related tasks, has been used to handle these problems. In this paper, we give an overview of the use of MTL in NLP tasks. We first review MTL architectures used in NLP tasks and categorize them into four classes, including the parallel architecture, hierarchical architecture, modular architecture, and generative adversarial architecture. Then we present optimization techniques on loss construction, data sampling, and task scheduling to properly train a multi-task model. After presenting applications of MTL in a variety of NLP tasks, we introduce some benchmark datasets. Finally, we make a conclusion and discuss several possible research directions in this field.

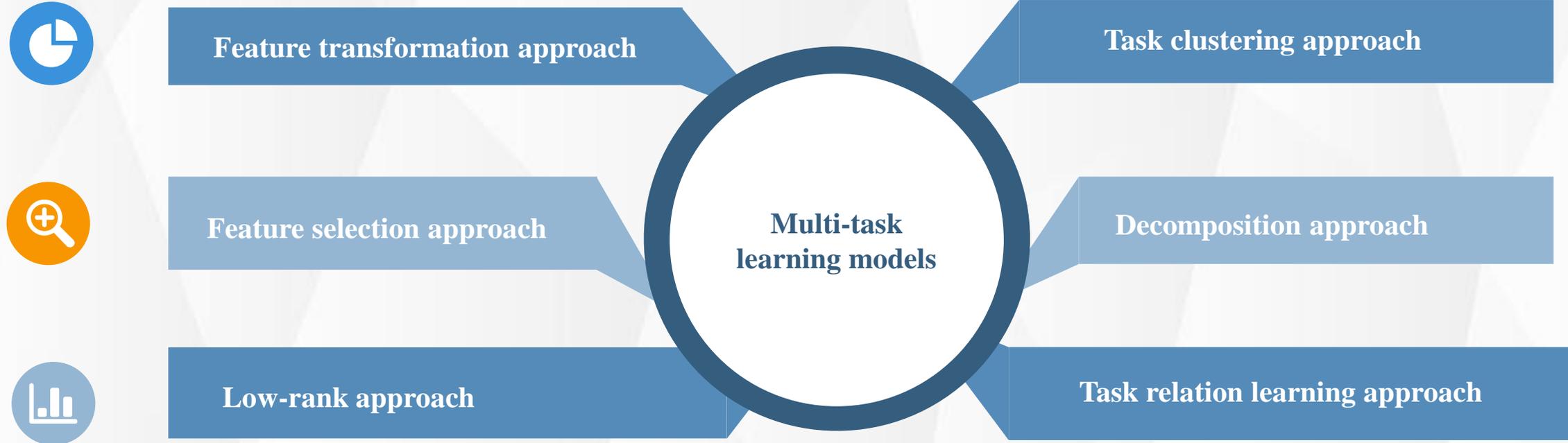
Subjects: **Artificial Intelligence (cs.AI)**

Cite as: [arXiv:2109.09138](https://arxiv.org/abs/2109.09138) [cs.AI]

(or [arXiv:2109.09138v1](https://arxiv.org/abs/2109.09138v1) [cs.AI] for this version)

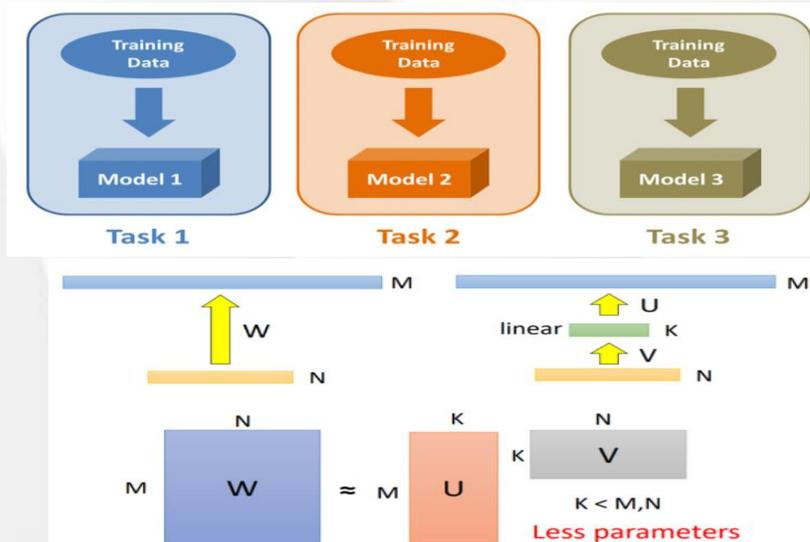
Shijie Chen, Yu Zhang, Qiang Yang. Multi-Task Learning in Natural Language Processing: An Overview. arXiv:2109.09138, 2021.

# Introduction



# Introduction

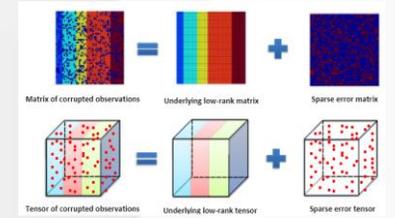
## Low-rank approach



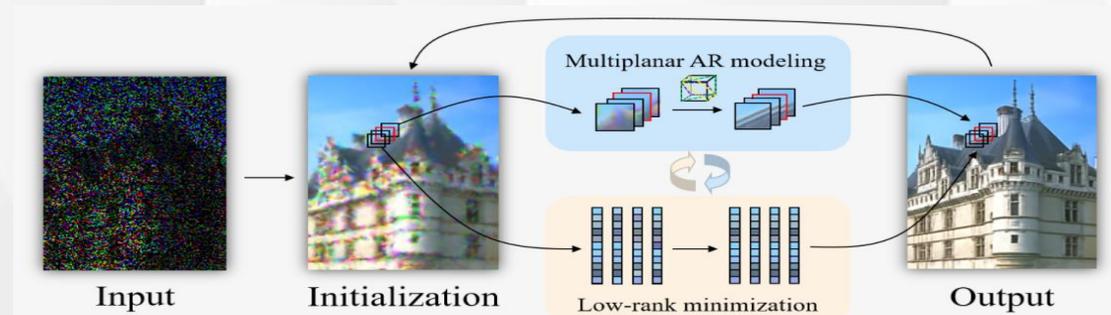
Relatedness among multiple tasks



Low-rank of parameters



Low-rank approach



# Introduction

## Low-rank approach

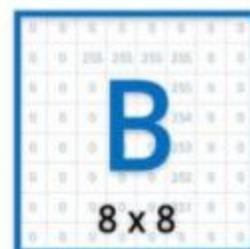
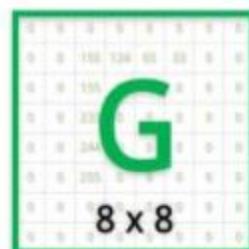
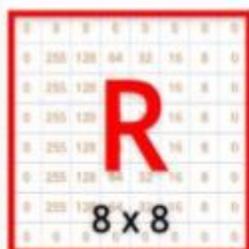
Matrix parameters



Matrix trace norm

Multi-task

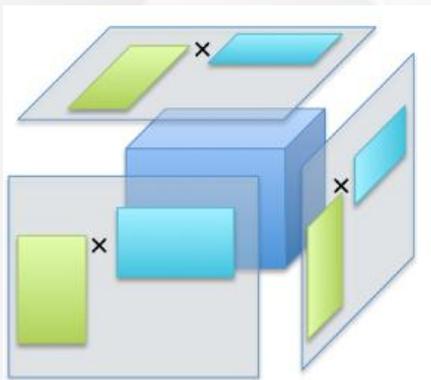
Image



Tensor trace norm

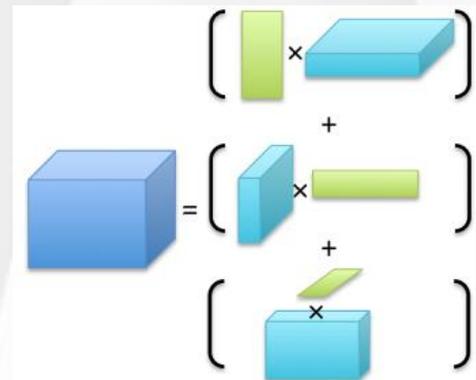
Multi-class classification

Overlapped tensor trace norms



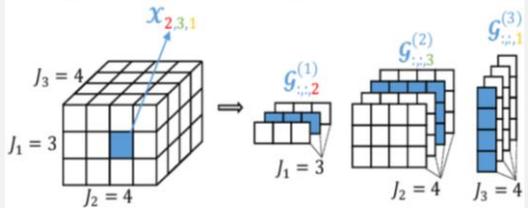
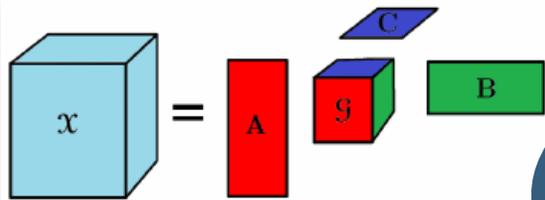
Tensor trace norm

Latent tensor trace norms

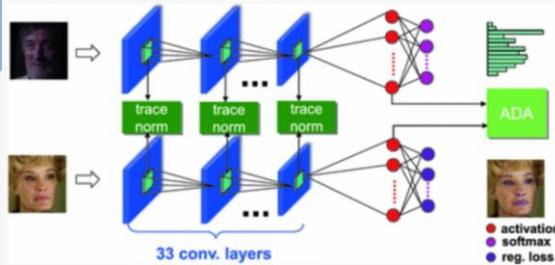


# Introduction

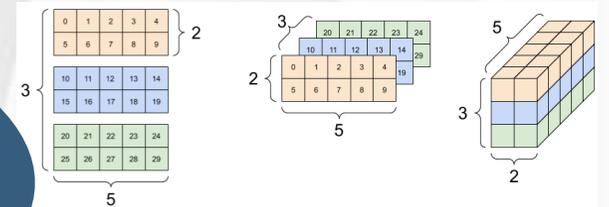
## Tucker Trace Norm



## Tensor-Train (TT) Trace Norm

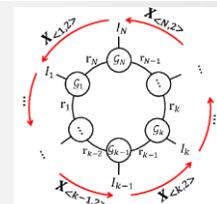


## LAF Trace Norm



## Overlapped tensor trace norms

## Tensor-Ring (TR) Trace Norm





# Multi-Task Learning via Generalized Tensor Trace Norm

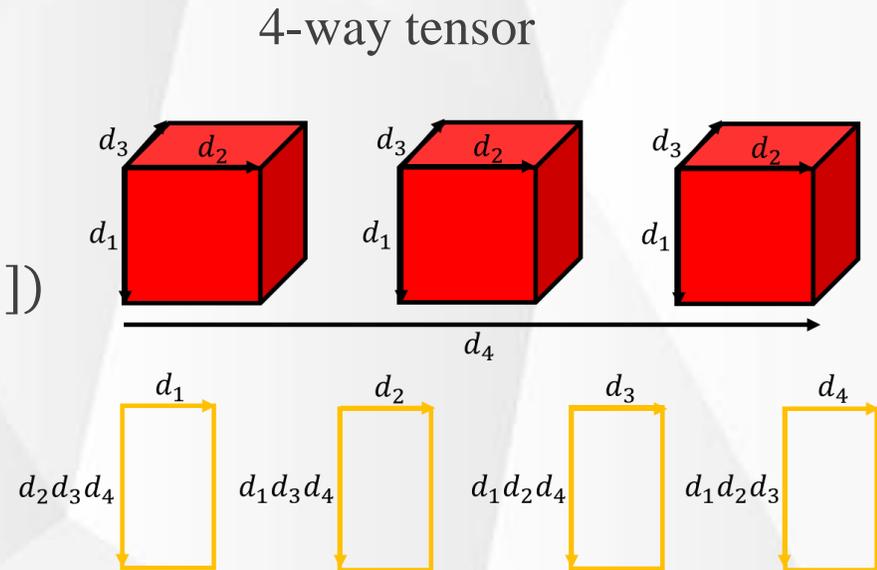
## 02 Existing tensor trace norms

# Existing tensor trace norms

## Tucker Trace Norm

$$\|\mathcal{W}\|_* = \sum_{i=1}^p \alpha_i \|\mathcal{W}_{(i)}\|_* \quad \mathcal{W} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

$$\text{s.t.} \begin{cases} \mathcal{W}_{(i)} := \text{reshape}(\text{permute}(\mathcal{W}, [i, 1, \dots, i-1, i+1, \dots, p]), [d_i, \prod_{j \neq i} d_j]) \\ \alpha_i \geq 0, \sum_{i=1}^p \alpha_i = 1 \end{cases}$$



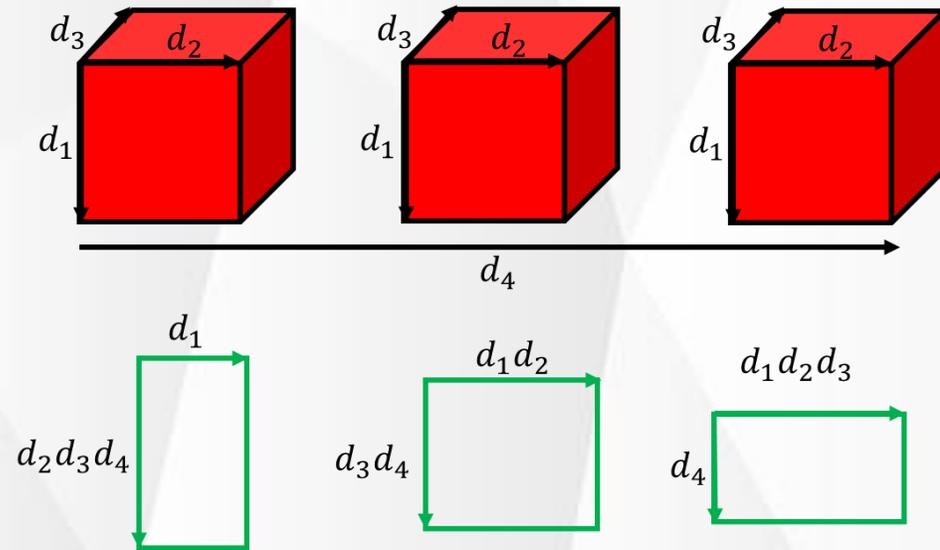
# Existing tensor trace norms

## TT Trace Norm

$$|||\mathcal{W}|||_* = \sum_{i=1}^{p-1} \alpha_i |||\mathcal{W}_{[i]}|||_* \quad \mathcal{W} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

$$\text{s.t.} \begin{cases} \mathcal{W}_{[i]} := \text{reshape}(\mathcal{W}, [\prod_{j=1}^i d_j, \prod_{j=i+1}^p d_j]) \\ \alpha_i \geq 0, \sum_{i=1}^p \alpha_i = 1 \end{cases}$$

4-way tensor



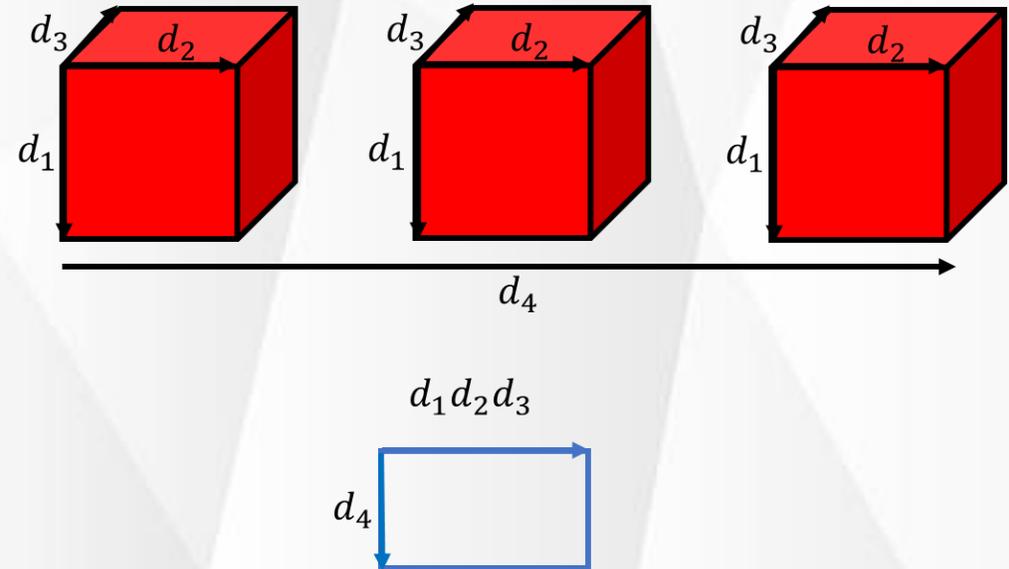
# Existing tensor trace norms

## LAF Trace Norm

$$|||\mathcal{W}|||_* = ||\mathcal{W}_{(p)}||_* \quad \mathcal{W} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

$$\text{s.t. } \mathcal{W}_{(p)} := \text{reshape}(\mathcal{W}, [\prod_{j=1}^{p-1} d_j, d_p])$$

4-way tensor



# Existing tensor trace norms

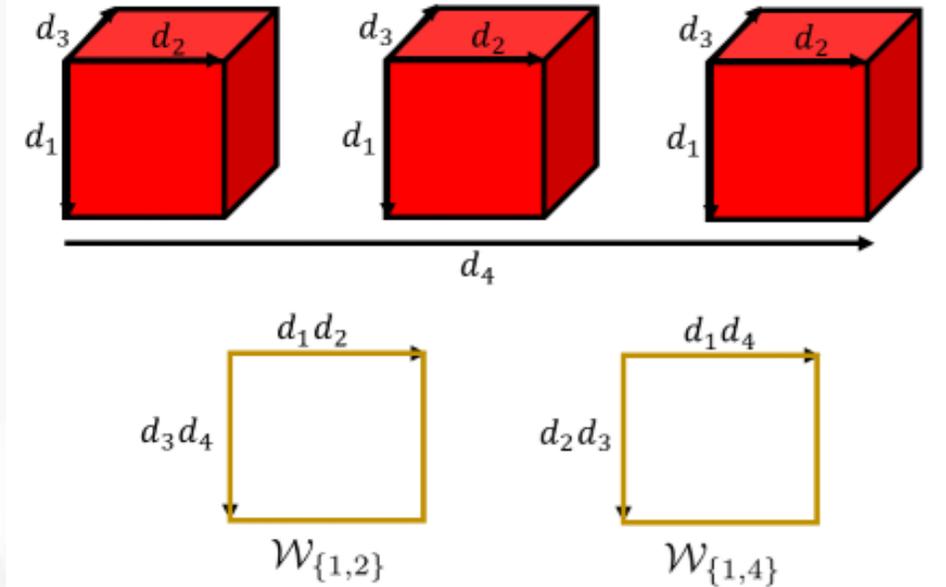
## TR trace norm

$$|||\mathcal{W}|||_* = \sum_{i=1}^p \alpha_i ||\mathcal{W}_{\langle i,d \rangle}||_*$$

$$\mathcal{W} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

$$\text{s.t.} \left\{ \begin{array}{l} \mathcal{W}_{\langle i,d \rangle} := \text{reshape}(\text{permute}(\mathcal{W}, [t, \dots, i, i+1, \dots, t-1]), [\prod_{j=t}^i d_j, \prod_{j=i+1}^{t+1} d_j]) \\ t = \begin{cases} i - d + 1 & \text{if } d \leq i \\ i - d + 1 + p & \text{otherwise} \end{cases} \\ \alpha_i \geq 0, \sum_{i=1}^p \alpha_i = 1 \end{array} \right.$$

4-way tensor





## **Multi-Task Learning via Generalized Tensor Trace Norm**

### **03 Generalized Tensor Trace Norm (GTTN)**

# Generalized Tensor Trace Norm (GTTN)

How to choose the way of tensor flattening?

- ◆ Try all possible ways of tensor flattening.



**Analysis on  
Existing  
Tensor Trace  
Norms**

Given the way of tensor flattening, how to determine the importance of resultant tensor flattenings?

Learn Weights:

- ◆ Variable
- ◆ Minimum
- ◆ Maximum
- ◆ Meta-learning



## Generalized Tensor Trace Norm (GTTN)

How to choose the way of tensor flattening?

→ Try all possible ways of tensor flattening

$$\mathbf{w}_* = \sum_{\mathbf{s}} \alpha_{\mathbf{s}} \|\mathbf{w}_{\{\mathbf{s}\}}\|_* \quad \mathbf{w} \in \mathbb{R}^{d_1 \times \dots \times d_p}$$

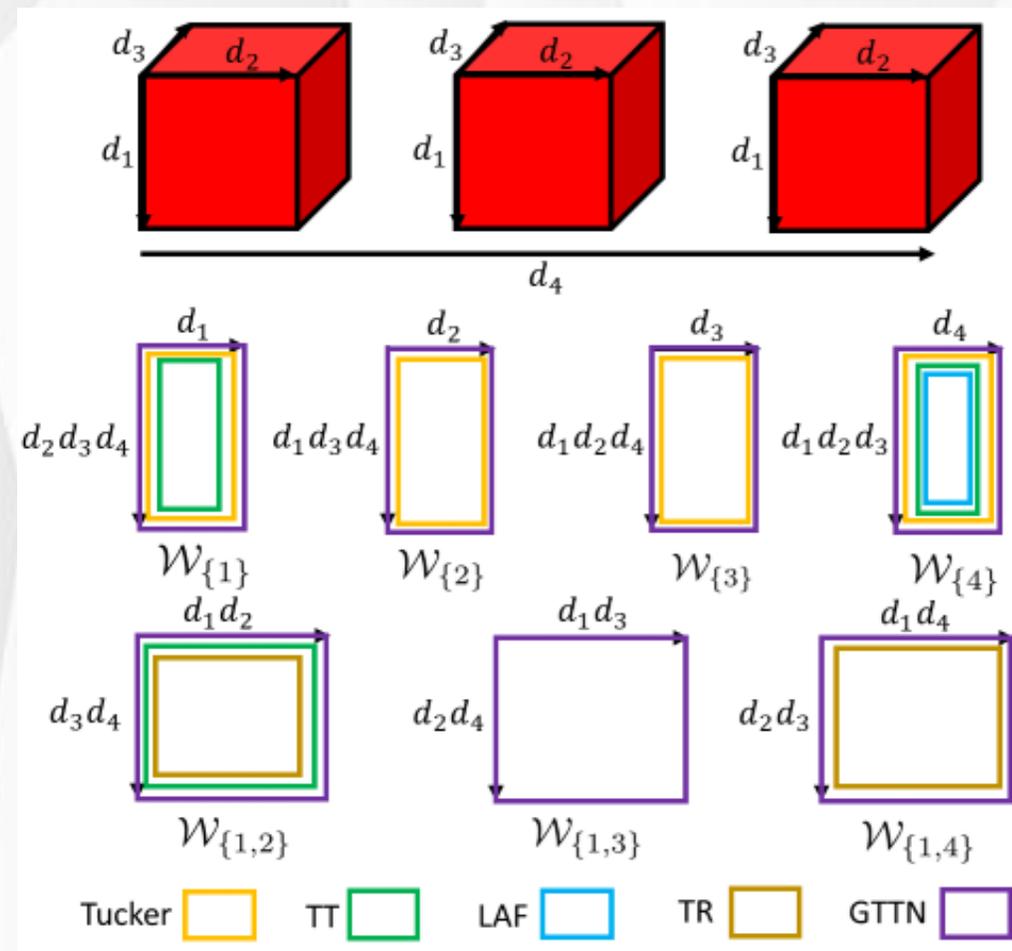
$$\text{s.t.} \begin{cases} \mathbf{w}_{\{\mathbf{s}\}} := \text{reshape}(\text{permute}(\mathbf{w}, [\mathbf{s}, \neg\mathbf{s}])), [\prod_{i \in \mathbf{s}} d_i, \prod_{j \in \neg\mathbf{s}} d_j] \\ \alpha_{\mathbf{s}} \geq 0, \sum_{\mathbf{s}} \alpha_{\mathbf{s}} = 1 \end{cases}$$

# Generalized Tensor Trace Norm (GTTN)

## How to choose the way of tensor flattening?

- ◆ Lemma 1: For a  $p$ -way tensor, There are  $2^{p-1} - 1$  distinct tensor flattening.
- ◆  $p \leq 5$
- ◆ # distinct tensor flattening  $\leq 15$

Comparison among the Tucker trace norm, TT trace norm, LAF trace norm, TR trace norm with  $d=2$ , and GTTN for a 4-way tensor.



# distinct tensor flattening: 4 3 1 2 7

# Generalized Tensor Trace Norm (GTTN)

How to determine  $\alpha$ ?



# Generalized Tensor Trace Norm (GTTN)

## Learning Weights

## Viewing $\alpha$ as variables to be optimized

$$\min_{\Theta, \alpha} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \Theta), y_j^i) + \lambda \sum_{s=1}^N \alpha_s \|\mathcal{W}_{\{s\}}\|_* \quad \text{s. t. } \alpha_s \geq 0, \sum_s \alpha_s = 1$$

$$\alpha_s = \frac{\exp\{\beta_s\}}{\sum_{t \in [p], t \neq \emptyset} \exp\{\beta_t\}}$$

softmax function

$\beta_s$  instead of  $\alpha_s$  is treated as a variable.

# Generalized Tensor Trace Norm (GTTN)

## Learning Weights

## Optimizing minimum matrix trace norm

$$\min_{\theta} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \theta), y_j^i) + \lambda \min_{\substack{s \subset [p] \\ s \neq \emptyset}} \|\mathcal{W}_{\{s\}}\|_*$$

Mathematically



Equivalent

$$\min_{\theta, \alpha} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \theta), y_j^i) + \lambda \sum_{i=1}^N \alpha_s \|\mathcal{W}_{\{s\}}\|_* \quad \text{s.t. } \alpha_s \geq 0, \sum_s \alpha_s = 1$$

softmax function

cannot achieve 0 or 1 exactly

# Generalized Tensor Trace Norm (GTTN)

## Learning Weights

## Optimizing maximum matrix trace norm

$$\min_{\Theta} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \Theta), y_j^i) + \lambda \max_{\substack{s \subseteq [p] \\ s \neq \emptyset}} \|\mathcal{W}_{\{s\}}\|_*$$

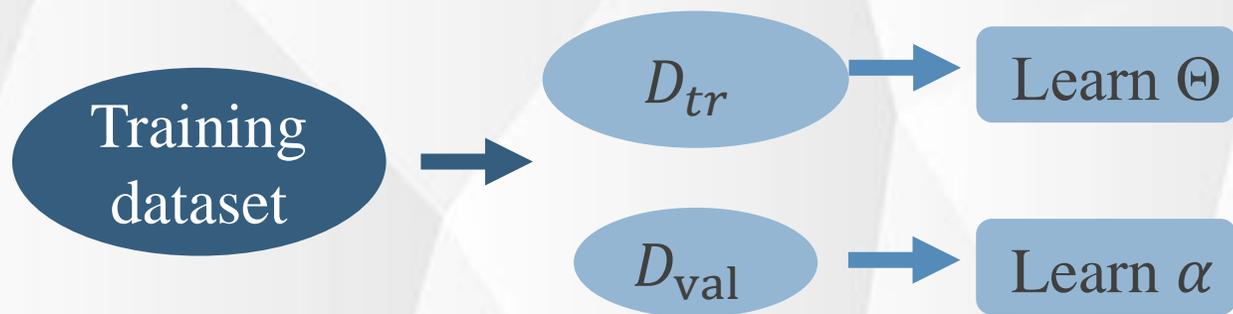
Pay attention to flattenings with the **largest** trace norm

Penalize all the matrix trace norm

# Generalized Tensor Trace Norm (GTTN)

Learning Weights

Meta-learning method



$$\min_{\alpha} \sum_{i=1}^m \frac{1}{|D_{val}^i|} \sum_{(x,y) \in D_{val}^i} l(f_i(\mathbf{x}; \Theta^*), y)$$

$$\text{s.t. } \Theta^* = \operatorname{argmin} \sum_{i=1}^m \frac{1}{|D_{tr}^i|} \sum_{(x,y) \in D_{tr}^i} l(f_i(\mathbf{x}; \Theta), y) + \lambda \sum_{s=1}^p \alpha_s \|\mathcal{W}_{\{s\}}\|_*$$

# Generalized Tensor Trace Norm (GTTN)

## Objective function

Given the Generalized Tensor Trace Norm (GTTN), the objective function of the deep multi-task model can be expressed as:

$$\min_{\Theta} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} l(f_i(x_j^i; \Theta), y_j^i) + \lambda \|\mathcal{W}\|_*$$

Empirical loss

Regularization term: GTTN

# Generalized Tensor Trace Norm (GTTN)

## Analysis

**THEOREM 2.** *For the solution  $\hat{\mathcal{W}}$  of problem (8) and  $\delta > 0$ , with probability at least  $1 - \delta$ , we have*

$$L(\hat{\mathcal{W}}) \leq \hat{L}(\hat{\mathcal{W}}) + \frac{2\rho\gamma C}{mn_0} \min_{\substack{s \neq \emptyset \\ s \subset [p]}} \left( \frac{\kappa m \sqrt{\ln d_s}}{\alpha_s n_0 d} + \frac{\ln d_s}{\alpha_s n_0} \right) + \sqrt{\frac{2}{m} \ln \frac{1}{\delta}}.$$



## **Multi-Task Learning via Generalized Tensor Trace Norm**

**04 Experiments**

# Experiments

## Baselines

DMTL	Deep Multi-Task Learning method
Tucker	The Tucker trace norm method
TT	The TT trace norm method
LAF	The LAF trace norm method
LAF- $i$	The trace norm regularization method based on the $i$ -th axis flattening
TR	TR trace norm method ( $d = 2$ )
LAF-TF	LAF tensor factorization method
Prod	The rank-product regularization method
GTTN-a	Setting the weights in GTTN to be same

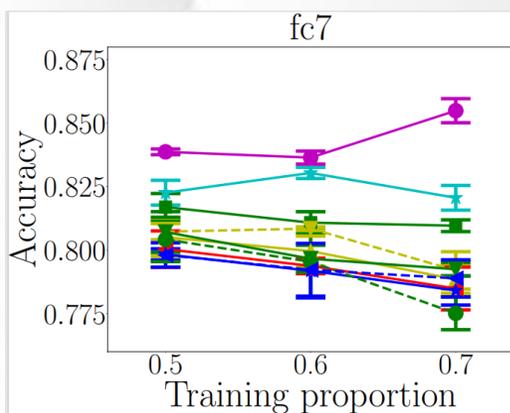
## Experiments

## Datasets

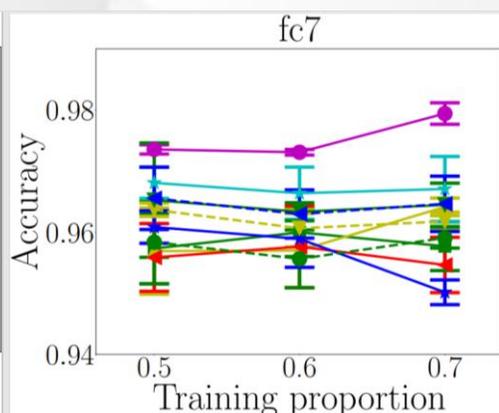
<b>Dataset</b>	<b>#Images</b>	<b>#Classes</b>	<b>#Tasks</b>
ImageCLEF	2,400	12	4
Office-Caltech	2,533	10	4
Office-31	4,110	31	3
Office-Home	15,500	65	4
DomainNet	600,000	345	6

# Experiments

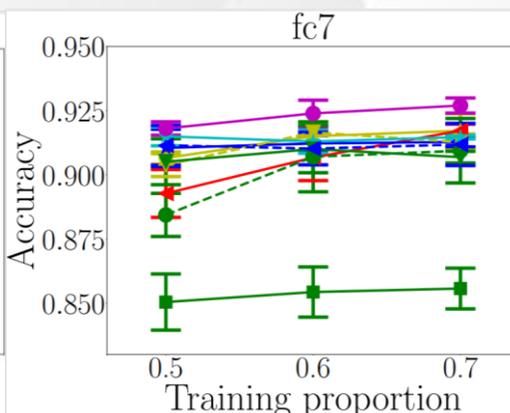
## Results: fc7 layer



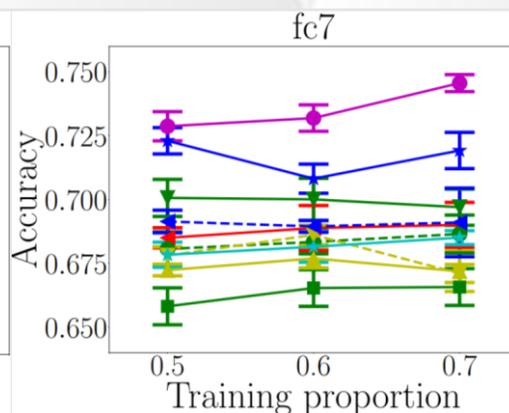
ImageCLEF



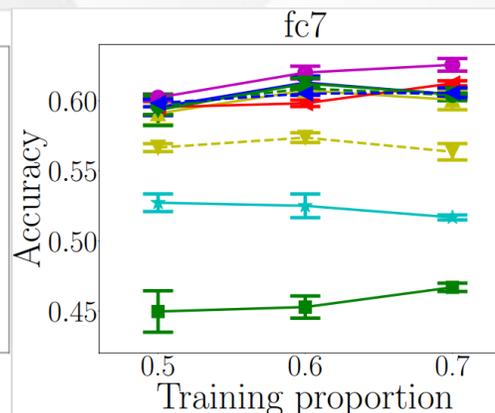
Office-Caltech



Office-31



Office-Home



DomainNet



DMTL



Tucker



TT



LAF



LAF-1



LAF-2



Prod



TR



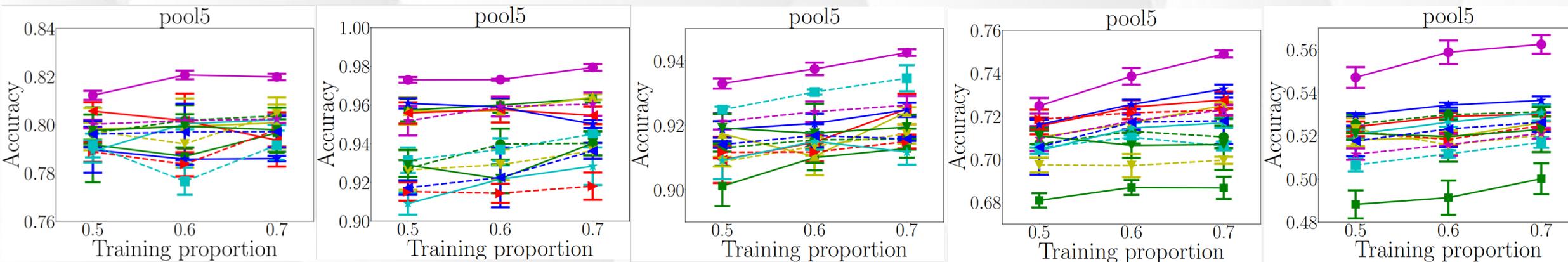
LAF-TF



GTTN

# Experiments

## Results: pool5 layer



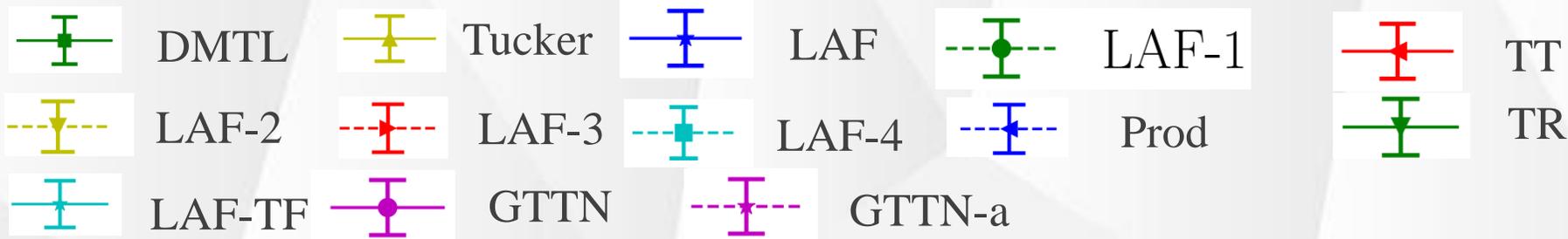
ImageCLEF

Office-Caltech

Office-31

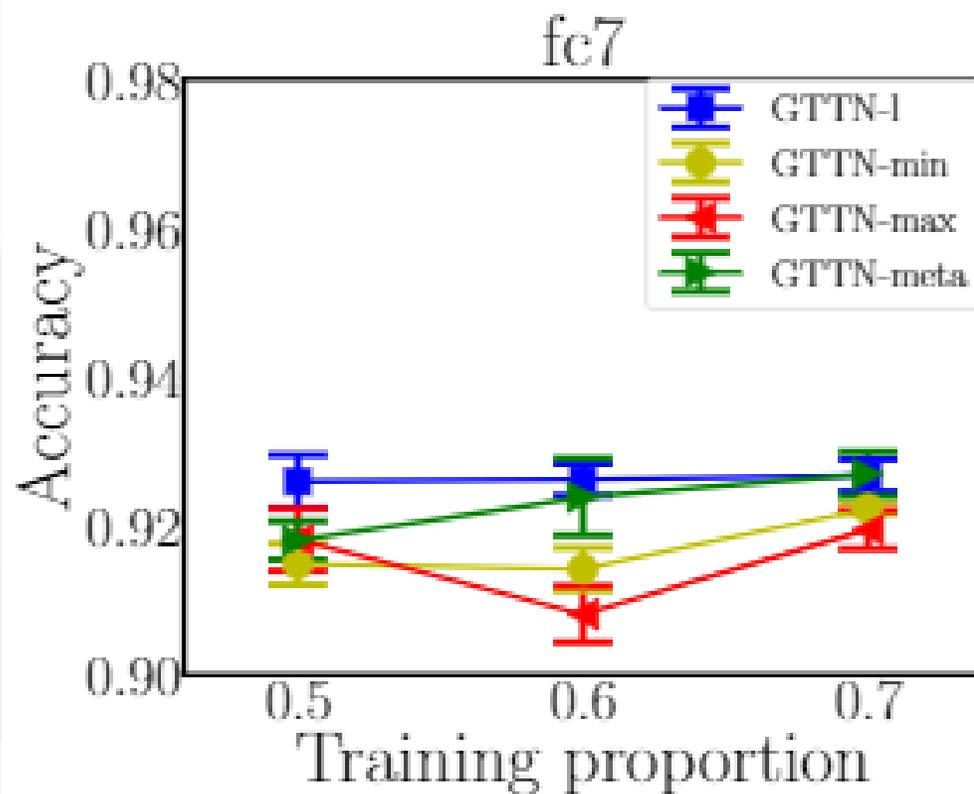
Office-Home

DomainNet

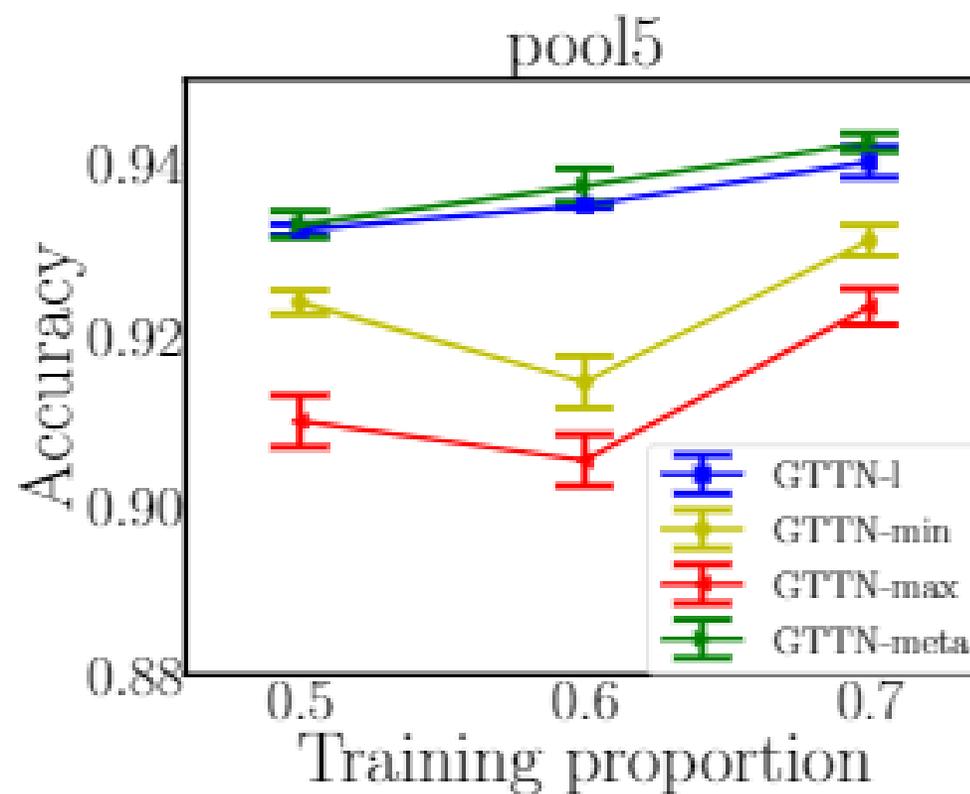


# Experiments

## Comparison on Strategies to Learn Weights



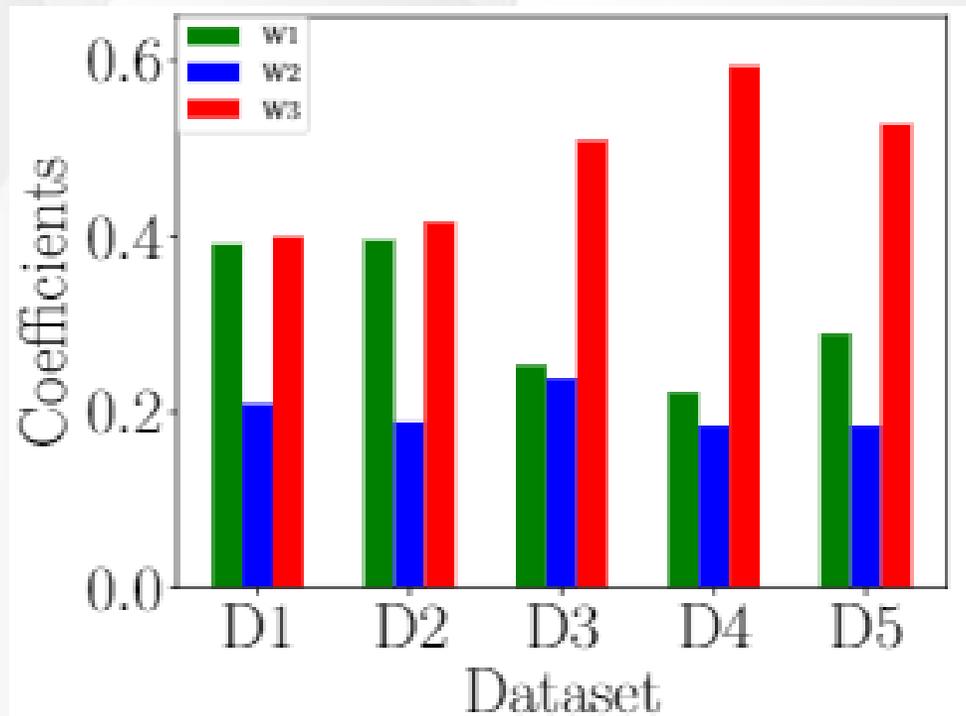
(a) Comparison of GTTN



(b) Comparison of GTTN

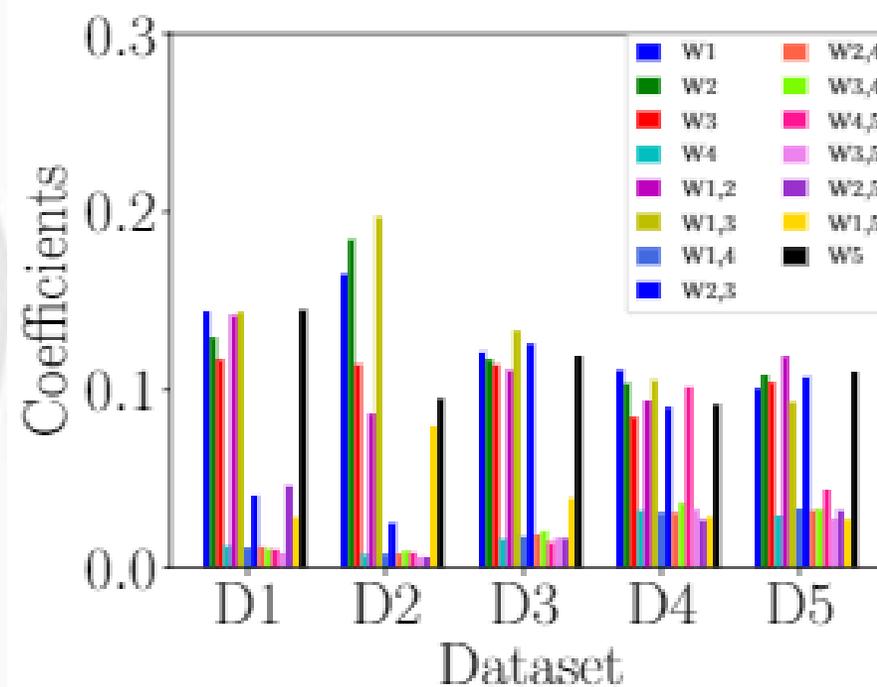
# Experiments

## Analysis on Learned Weights



(c) Learned  $\alpha$  (fc7)

$w_{\{2\}}$  is smaller

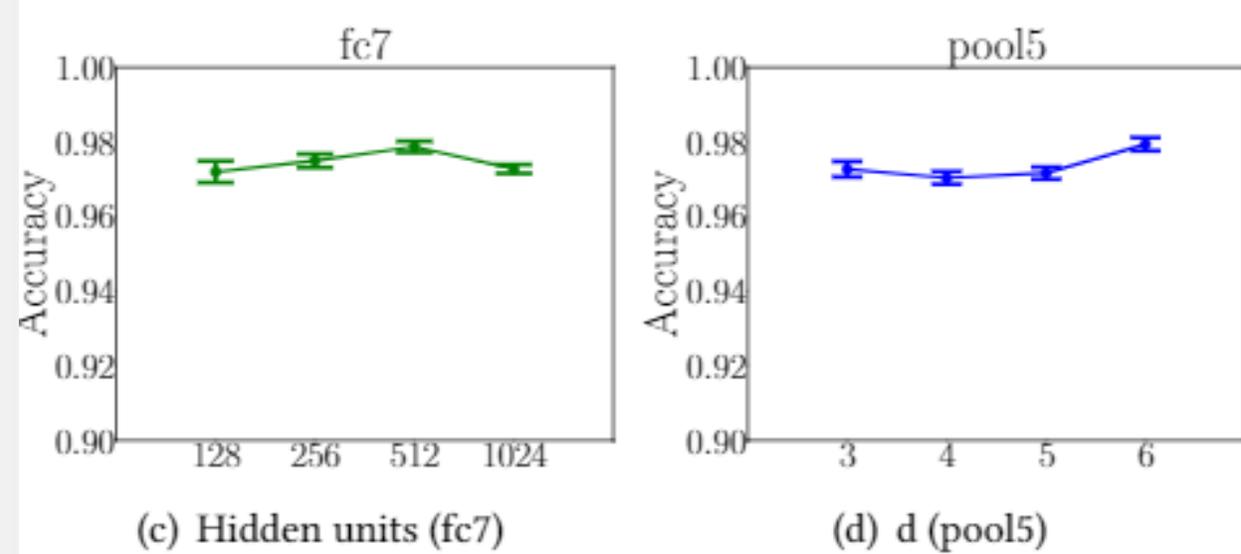
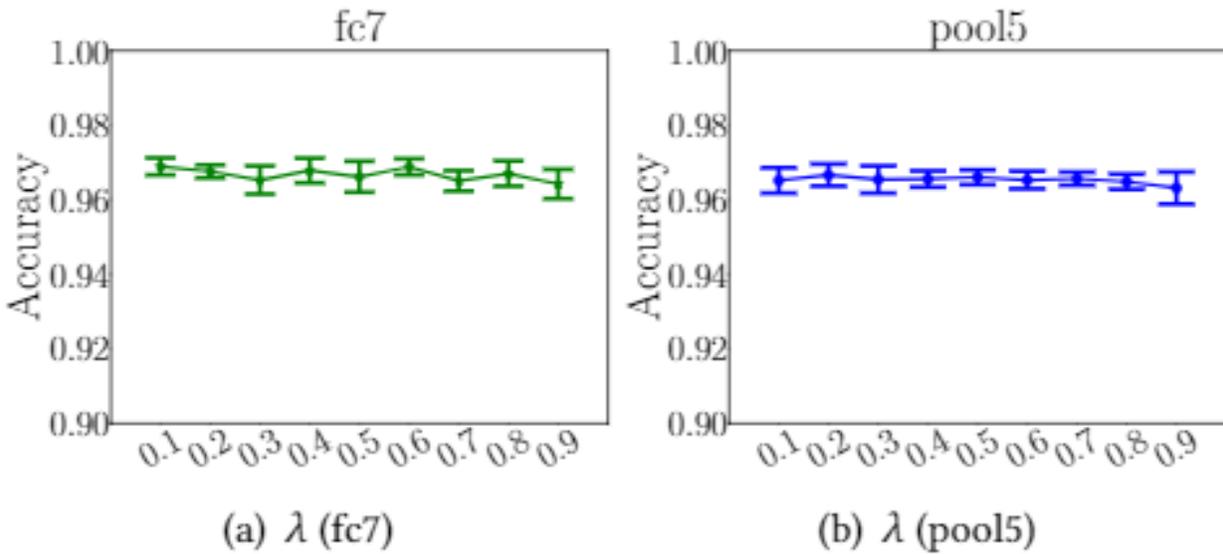


(d) Learned  $\alpha$  (pool5)

$w_{\{1\}}, w_{\{2\}}, w_{\{3\}}, w_{\{1,2\}}, w_{\{1,3\}}, w_{\{5\}}$  are larger

# Experiments

## Sensitivity Analysis



The performance is **not sensitive** to  $\lambda$

number of hidden units = 512,  $d=6$

## Conclusion

- The generalized tensor trace norm (GTTN) to capture all the low-rank is effective.
- Learning weights of each tensor flattening to identify the importance of each structure is helpful.
- The GTTN method performs better than baseline methods.



# Thank you !

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